

Decision Problems in Semigroups

Automaton, automatic, FA-presentable: decision problems for three automata-theoretic approaches to semigroups

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An automaton semigroup is defined as the semigroup generated by the action of a Mealy automaton. An automatic structure for a semigroup (a concept which emerged in the late 1990s as a generalization of automatic structures for groups as studied by Epstein et al.) describes a semigroup in terms of a regular language of normal forms and synchronous rational relations that describe right-multiplication of elements by generators in terms of those normal forms. An FA-presentation (also called an automatic presentations, a concept developed by Khoushainov & Nerode as a means of extending finite model theory to infinite structures while maintaining decidability of interesting problems) describes a relational structure in terms of a regular language of abstract representatives for its elements (not necessarily words over a generating set) and synchronous rational relations describing each of its relations in terms of those representatives (viewing n -ary operations as $(n + 1)$ -ary relations).

This talk will be a brief "high-level" survey, comparing and contrasting these three ways of describing and computing with semigroups, with a particular focus on results on decision problems and undecidability results.