

Decision Problems in Semigroups

How should we define the word problem of a semigroup?

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The word problem of an algebraic structure (with respect to a generating set X) is the problem of deciding, given two expressions in elements of X and their inverses, whether they both represent the same element of the structure. For groups, it suffices to be able to decide which words represent the identity element, so the word problem of a group G with respect to a (finite) generating set X is classically defined as $W(G, X) = \{w \mid w =_G 1\}$. For semigroups, this would not encapsulate the necessary information even in the case that an identity element is present, so we need another approach. In this talk I will discuss the advantages and disadvantages of the two main approaches to the semigroup word problem that have been used so far: the ‘unfolded’ and ‘two-tape’ word problems, as well as whether, especially for certain specific classes of semigroups, there might be other approaches that would be more suitable.