

Session: Identities in combinatorial monoids

Identities and bases in the sylvester and Baxter monoids

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The ubiquitous plactic monoid [1], also known as the monoid of Young tableaux, has deep connections to several areas of mathematics, in particular, to the theory of symmetric functions. An actively-studied problem concerns the identities satisfied by the plactic monoids of finite rank: it is known that these satisfy non-trivial identities dependent on rank [2], but that there is no “global” identity satisfied independently of rank [3]. In contrast, monoids related to the plactic monoid, such as the hypoplactic monoid (the monoid of quasi-ribbon tableaux, connected with quasisymmetric functions), sylvester monoid (the monoid of binary search trees) and Baxter monoid (pairs of twin binary search trees, connected with Baxter), satisfy global identities, and the shortest identities have been characterized [4].

This talk will focus on results on the sylvester and Baxter monoids, obtained in joint work [5] with Alan Cain and António Malheiro (FCT NOVA), and following on the results obtained by the same authors on the hypoplactic monoid [6]. We show how to embed the sylvester monoid of higher rank into a direct product of copies of the sylvester monoid of rank 2. As such, we show that these monoids satisfy exactly the same identities, for which we give a full description. Then, we show that the variety generated by the sylvester monoid has finite axiomatic rank, and give a finite basis for its equational theory. Finally, we show parallel results obtained for the Baxter monoid.

References

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