

Sessão: Métodos Categoriais em Álgebra e Topologia

From compact metric spaces to metric compact spaces

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An important source of inspiration for our research over the past years has been the work [3] of Nachbin about topological spaces equipped with an additional partial order relation, subject to a compatibility condition. Our second main motivation stems from [2] where metric spaces are studied as categories enriched in the quantale $[0, \infty]$. Combining these ideas, and motivated by [5], we started to investigate *metric compact Hausdorff spaces* which, by definition, are metric spaces equipped with a compatible compact Hausdorff topology. By design, this notion generalises Nachbin's partially ordered compact spaces; but also classical compact metric spaces can be viewed as metric compact Hausdorff spaces. In this talk we give an overview of results about completeness properties of this kind of metric spaces, generalisations of the Vietoris topology [6] and the “Pompeiu-Hausdorff” metric [1, 4], and metric versions of Stone-type dualities.

References

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