

## FRACTIONAL RESPONSES WITH SPATIAL DEPENDENCE

LUÍS SILVEIRA SANTOS

ISEG – School of Economics and Management, Universidade de Lisboa; REM – Research in  
Economics and Mathematics, CEMAPRE

ISEG – School of Economics and Management, Rua do Quelhas n.º 6, 1200-781 Lisboa, Portugal

[lsantos@iseg.ulisboa.pt](mailto:lsantos@iseg.ulisboa.pt)

### **Abstract:**

This paper introduces a specification to estimate models for spatially dependent fractional responses with additive errors. This specification, the Fractional Spatial Lag model (FSLM), extends the approach of Papke and Wooldridge (1996), incorporating spatial dependence through the dependent variable. Moreover, it allows to handle observations at the boundaries, zero and one. Estimation is addressed by the Generalized Method of Moments, with well-known instruments. A second specification is introduced, based on a series expansion of the FSLM around the spatial lag parameter equal to zero, the approximated FSLM. This procedure allows to obtain an approximated reduced form for the spatially dependent fractional response, simplifying the estimation of the partial effects. The approximated FSLM is also estimated by the Generalized Method of Moments. Robust inference is standard. Closed formula expressions for the partial effects are deduced for both models. An extensive Monte Carlo simulation study is presented to investigate the finite sample properties of the estimators for the two approaches and the corresponding partial effects. Experiments show that the estimators for the spatial lag parameter and the regression coefficients perform well in terms of bias and root mean square error for a great variety of sampling designs. However, the estimates for the spatial lag parameter on the approximated FSLM exhibit a small bias when the true value is close to one, as expected. Nevertheless, the estimation of the partial effects is much less affected.

**Keywords:** Spatial Econometrics, Fractional responses, Generalized Method of Moments, Partial effects

**JEL Classification:** C13, C21, C25

**Acknowledgments:** The author gratefully acknowledge the Portuguese Foundation for Science and Technology (Fundação para a Ciência e a Tecnologia) for the financial support through national funds (CEMAPRE/REM - UIDB/05069/2020)