

On a penalty function in the Erlang renewal dual risk model under independent randomised observations

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We consider the dual risk model with financial application, where the random gains (or claims, using the primal insurance risk model) occur under a renewal process. Due to the mathematical robustness of the model we do not need to impose the usual economic condition, as shown by [2], despite the fact that the ruin probability being one when the condition is not respected. We introduce a Gerber-Shiu type of penalty function applied to the dual model and consider that randomised observations are set in place as [1] do for the primal or classical compound Poisson insurance risk model. We go further than these authors by studying a renewal risk process, mention in particular the Erlang(n) renewal model, and an independent observational Poisson process. Under these model ruin can only arise if it is indeed observed, we mean, the risk process may cross downwards the zero but ruin may not happen if it recovers before it can be observed. We develop integral and differential equations, from which we study solutions for some cases, and particularly solve numerically for some examples in full. We show some figures and graphs from some chosen numerical examples. We compare them, where possible, with those of [1] although worked for the primal compound Poisson risk model. Also with those of [3].

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