Teoria dos Operadores e Aplicações

Quaternionic numerical range

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The numerical range $W(T)$ of a bounded linear operator T on a complex Hilbert space $\mathcal{H}$ is the range of the unit circle of $\mathcal{H}$ under the quadratic map $x \mapsto \langle Tx, x\rangle$ induced by T

$$W(T) = \{\langle Tx, x\rangle : \|x\| = 1, x \in \mathcal{H}\} \subset \mathbb{C}.$$

One motivation in studying the numerical range of T is that it gives an estimate of the location of the spectrum $\sigma(T)$, the set of scalars λ for which $T - \lambda I$ is not invertible. In fact, $\sigma(T) \subset \overline{W(T)}$.

In 1951 Kippenhahn introduced the study of the numerical range for quaternionic operators over finite-dimensional Hilbert spaces. A striking difference between the quaternionic and complex numerical ranges is convexity. Contrary to what happens in the complex case, the quaternionic numerical range is not convex in general.

While earlier studies on the quaternionic numerical range have been carried out in finite dimension by the work of Kippenhahn, Au-Yeung , So and Thompson and, more recently, some researchers in this team [1]-[6], there is a lack of results in infinite-dimensional quaternionic Hilbert spaces. A possible explanation for this is the nonexistence, until recently, of a suitable notion of spectrum in the quaternionic setting. In 2006, Colombo and Sabadini proposed the notion of S-spectrum, which lead to spectral theorems for quaternionic normal operators.

In my talk I will present some of the problems that together with my colleagues Cristina Diogo, Luís Carvalho and Sérgio Mendes we have addressed and the results obtained so far.

References

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