

Monoids and Combinatorics

Conjugacy growth in polycyclic monoids

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The *conjugacy growth function* of a finitely generated group G counts the number of conjugacy classes intersecting the ball of radius n in the Cayley graph of G centered at the identity, for all $n \geq 0$. It has been studied for free groups [16, 10, 17], hyperbolic groups [11, 12], solvable groups [5], linear groups in [6], acylindrically hyperbolic groups [15, 1], certain branch groups [13], and several other classes of groups [14]. In the last few years, the *conjugacy growth series* (the generating series associated with the conjugacy growth functions) have been computed for several classes of groups based on the description of sets consisting of minimal length representatives from all conjugacy classes [9, 8, 1, 7].

Given a conjugacy defined for monoids, the conjugacy growth series of the groups can be extended to the monoids. The purpose of this talk is to give a brief overview of the conjugacy growth series of the polycyclic monoids, for the conjugacies \sim_n , \sim_p , and \sim_c .

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