On some nonlinear parabolic equations with non-monotone multivalued terms

We study the existence of solutions to the initial-boundary value problem for the following parabolic differential inclusion in $Q_T := \Omega \times [0,T]$:

$$\left(\frac{\partial}{\partial t}\right) u(x,t) - \Delta_p u(x,t) \in -\partial\phi(u(x,t)) + G(x,t,u(x,t)),$$

where Ω is a bounded open subset of $\mathbb{R}^N$ with smooth boundary $\partial\omega$, $T > 0$, Δ_p is the p-Laplace differential operator, $\partial\phi$ denotes the subdifferential of a proper lower semicontinuous convex function $\phi : \mathbb{R} \to [0,\infty]$ and $G : Q_T\mathbb{R} \to 2^{\mathbb{R}}$ is a nonmonotone multivalued mapping. We firstly set up a framework which enables us to treat wider nonlinearity of $G(\cdot,\ldots,u)$, more precisely, to cover the growth condition on $G(\cdot,\ldots,u)$ up to the Sobolev-subcritical range, and secondly to adapt and improve the techniques and arguments developed in [OtaniStaicu1] and [OtaniStaicu2], in order to obtain existence results for the initial boundary value problem to parabolic inclusion generalizing corresponding results given by many authors, especially given in [Otani1], [Otani2] and [OtaniStaicu2] where the semi-linear case $p = 2$ is considered. We give two types of local existence results for the cases where $G(\cdot,\ldots,u)$ is upper semicontinuous and lower semicontinuous, and also discuss the extension of large or small local solutions along the lines of arguments developed in [Otani1]. This is a joint work with Mitsuharu Otani from Waseda University, Japan.

Vasile Staicu, Universidade de Aveiro, CIDMA